

$$I = \int e^{\sqrt{x}} dx, \quad \sqrt{x} = t \text{ とおく}$$

$$x = t^2 \text{ より } \frac{dx}{dt} = 2t.$$

$$\text{よって } dx = 2t \cdot dt \text{ より}$$

$$I = \int e^t \cdot 2t \, dt$$

$$= 2 \int (e^t)' t \, dt$$

$$= 2 \left(e^t \cdot t - \int e^t \cdot 1 \, dt \right)$$

$$= 2(t e^t - e^t) + C$$

$$= 2(t-1)e^t + C$$

$$= 2(\sqrt{x}-1)e^{\sqrt{x}} + C \quad (C \text{ は定数}).$$