

$$\begin{aligned}
 (1) \quad \int \frac{x}{\sqrt{x+1}+1} dx &= \int \frac{x(\sqrt{x+1}-1)}{(\sqrt{x+1}+1)(\sqrt{x+1}-1)} dx \\
 &= \int \frac{x(\sqrt{x+1}-1)}{(x+1)-1} dx \\
 &= \int (\sqrt{x+1}-1) dx \\
 &= \frac{2}{3}(x+1)\sqrt{x+1} - x + C \quad (C \text{ は積分定数})
 \end{aligned}$$

(2)

$$I = \int e^{ax} \cos x dx \quad a \neq 0$$

$$I = \int \left(\frac{1}{a} e^{ax} \right)' \cos x dx$$

$$= \frac{1}{a} e^{ax} \cos x - \int \frac{1}{a} e^{ax} (-\sin x) dx$$

$$= \frac{1}{a} e^{ax} \cos x + \frac{1}{a} \int e^{ax} \sin x dx$$

$$= \frac{1}{a} e^{ax} \cos x + \frac{1}{a} \int \left(\frac{1}{a} e^{ax} \right)' \sin x dx$$

$$= \frac{1}{a} e^{ax} \cos x + \frac{1}{a^2} e^{ax} \sin x - \frac{1}{a^2} \int e^{ax} \cos x dx$$

$$\text{よって } I = \frac{1}{a^2} e^{ax} (a \cos x + \sin x) - \frac{1}{a^2} I \quad (*)$$

$$\left(1 + \frac{1}{a^2}\right) I = \frac{1}{a^2} e^{ax} (a \cos x + \sin x) + C' \quad (C' \text{ は定数})$$

$$\frac{a^2+1}{a^2} I = \frac{1}{a^2} e^{ax} (a \cos x + \sin x) + C'$$

$$\text{よって } \boxed{I = \frac{1}{a^2+1} e^{ax} (\sin x + a \cos x) + C \quad (C \text{ は定数})}$$

$$(8) \text{ (7)} \quad I = \int e^{ax} \cos x dx \quad a \neq 0$$

$$I = \int e^{ax} (\sin x)' dx$$

$$= e^{ax} \sin x - \int a e^{ax} \sin x dx$$

$$= e^{ax} \sin x - \int a e^{ax} (-\cos x)' dx$$

$$= e^{ax} \sin x + a e^{ax} \cos x + \int a^2 e^{ax} (-\cos x) dx$$

$$= e^{ax} (\sin x + a \cos x) - a^2 I + C'' \quad (C'' \text{ は定数})$$

$$\text{よって } (1+a^2)I = e^{ax} (\sin x + a \cos x) + C'' \quad (*)$$

$$\boxed{I = \frac{1}{a^2+1} e^{ax} (\sin x + a \cos x) + C \quad (C \text{ は定数})}$$

(別解2)

$$(e^{ax} \sin x)' = a e^{ax} \sin x + e^{ax} \cos x \quad \text{--- ①}$$

$$(e^{ax} \cos x)' = -e^{ax} \sin x + a e^{ax} \cos x \quad \text{--- ②}$$

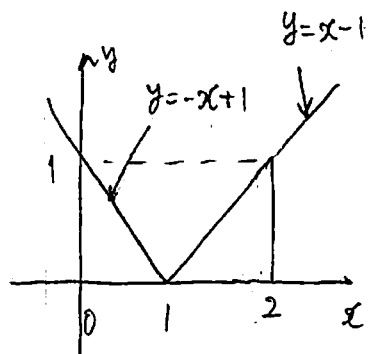
$$\text{①} + \text{②} \times a \text{ より } \{ e^{ax} (\sin x + a \cos x) \}' = (1 + a^2) e^{ax} \cos x$$

$$\therefore e^{ax} \cos x = \left\{ \frac{1}{1+a^2} e^{ax} (\sin x + a \cos x) \right\}' \text{ だから}$$

$$\boxed{\int e^{ax} \cos x \, dx = \frac{1}{1+a^2} e^{ax} (\sin x + a \cos x) + C}$$

(Cは定数)

(3)



$$\begin{aligned}
 & \int_0^2 |x-1| dx \\
 &= \int_0^1 (-x+1) dx + \int_1^2 (x-1) dx \\
 &= \left[-\frac{x^2}{2} + x \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^2 \\
 &= -\frac{1}{2} + 1 + \frac{4}{2} - 2 - \frac{1}{2} + 1 \\
 &= \frac{1}{2} + \frac{1}{2} = \boxed{1}
 \end{aligned}$$