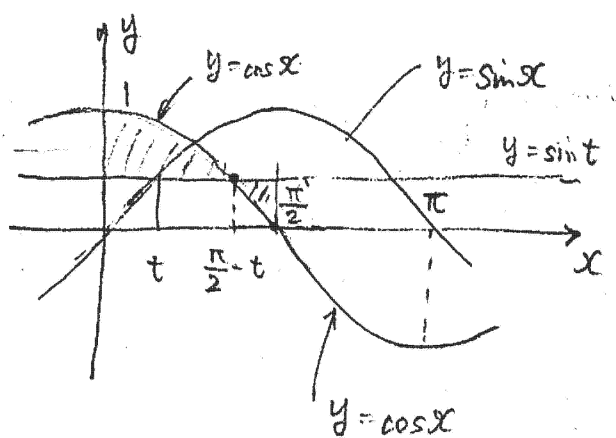




$$(1) t = \frac{\pi}{6} \text{ のとき } \sin \frac{\pi}{6} = \frac{1}{2} \text{ より}$$

$$\cos x - \sin t = 0 \text{ をとくと}$$

$$\cos x - \frac{1}{2} = 0 \text{ より}$$

$$\cos x = \frac{1}{2}$$

$$\therefore x = \frac{\pi}{3}$$

$$(2) f\left(\frac{\pi}{6}\right) = \int_0^{\frac{\pi}{3}} \left(\cos x - \frac{1}{2}\right) dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(-\cos x + \frac{1}{2}\right) dx$$

$$= \left[\sin x - \frac{1}{2}x\right]_0^{\frac{\pi}{3}} + \left[-\sin x + \frac{1}{2}x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{\sqrt{3}}{2} - \frac{\pi}{6} - 1 + \frac{\pi}{4} + \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

$$= \sqrt{3} - 1 + \frac{\pi}{4} - \frac{\pi}{3}$$

$$= \boxed{\sqrt{3} - 1 - \frac{\pi}{12}}$$

$$(3) \cos x = \sin t \text{ より}$$

$$\cos x = \cos\left(\frac{\pi}{2} - t\right)$$

$$\therefore x = \frac{\pi}{2} - t + 2n\pi \text{ または } x = -\left(\frac{\pi}{2} - t\right) + 2n\pi$$

(n は整数) とあるが

$$0 \leq x \leq \frac{\pi}{2} \text{ より } x = \frac{\pi}{2} - t$$

$$(4) f(t) = \int_0^{\frac{\pi}{2}-t} (\cos x - \sin t) dx + \int_{\frac{\pi}{2}-t}^{\frac{\pi}{2}} (-\cos x + \sin t) dx$$

$$= \left[\sin x - (\sin t) \cdot x\right]_0^{\frac{\pi}{2}-t} + \left[-\sin x + (\sin t) \cdot x\right]_{\frac{\pi}{2}-t}^{\frac{\pi}{2}}$$

$$= \sin\left(\frac{\pi}{2} - t\right) - (\sin t)\left(\frac{\pi}{2} - t\right) - 1 + \sin\left(\frac{\pi}{2} - t\right) + (\sin t)\frac{\pi}{2} - (\sin t)\left(\frac{\pi}{2} - t\right)$$

$$= 2\sin\left(\frac{\pi}{2} - t\right) - 1 + (\sin t)\left\{-\frac{\pi}{2} + t\right\} \times 2 + \frac{\pi}{2}$$

$$= \boxed{2\cos t - 1 + \left(2t - \frac{\pi}{2}\right)\sin t}$$

$$(5) f'(t) = -2\sin t + 2\sin t + \left(2t - \frac{\pi}{2}\right)\cos t = \frac{4t - \pi}{2} \cos t$$

よって増減表は以下のようになる

t	0	...	$\frac{\pi}{4}$	...	$\frac{\pi}{2}$
f'(t)		-	0	+	
f(t)		↘	最小	↗	

よって $t = \frac{\pi}{4}$ で f(t) は最小値をとる

$$\text{その値は } f\left(\frac{\pi}{4}\right) = 2 \times \frac{\sqrt{2}}{2} - 1 + \left(2 \times \frac{\pi}{4} - \frac{\pi}{2}\right) \times \frac{\sqrt{2}}{2}$$

$$= \boxed{\sqrt{2} - 1} \text{ ととる}$$