

$$\begin{aligned}
\int_0^{\frac{\pi}{4}} \frac{x}{\cos^2 x} dx &= \int_0^{\frac{\pi}{4}} x (\tan x)' dx \\
&= [x \tan x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 1 \times \tan x dx \\
&= \frac{\pi}{4} \times 1 - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx \\
&= \frac{\pi}{4} + [\log |\cos x|]_0^{\frac{\pi}{4}} \\
&= \frac{\pi}{4} + \log \left| \cos \frac{\pi}{4} \right| - \log |\cos 0| \\
&= \frac{\pi}{4} + \log \left(\frac{1}{\sqrt{2}} \right) - \log 1 \\
&= \frac{\pi}{4} + \log 2^{-\frac{1}{2}} \\
&= \boxed{\frac{\pi}{4} - \frac{1}{2} \log 2}
\end{aligned}$$