(i) $\displaystyle\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{(\cos x)'}{\cos x}\,dx = \boxed{-\log|\cos x| + C}$

(ii) $\displaystyle I(\theta) = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}-\theta} \tan x\,dx = \left[-\log|\cos x|\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}-\theta}$

$$= -\log\left|\cos\left(\frac{\pi}{2}-\theta\right)\right| + \log\left|\cos\frac{\pi}{4}\right|$$

$$= -\log|\sin\theta| + \log\left(\frac{1}{\sqrt{2}}\right)$$

$$= \log 2^{-\frac{1}{2}} - \log|\sin\theta|$$

$$= -\frac{1}{2}\log 2 - \log|\sin\theta|$$

よって $\displaystyle L = \lim_{\theta\to+0}\left(I(\theta) - I(2\theta)\right)$

$$= \lim_{\theta\to+0}\left\{\left(-\frac{1}{2}\log 2 - \log|\sin\theta|\right) - \left(-\frac{1}{2}\log 2 - \log|\sin 2\theta|\right)\right\}$$

$$= \lim_{\theta\to+0}\left(\log|\sin 2\theta| - \log|\sin\theta|\right)$$

$$= \lim_{\theta\to+0}\log\left|\frac{2\sin\theta\cos\theta}{\sin\theta}\right| = \lim_{\theta\to+0}\log|2\cos\theta| = \boxed{\log 2}$$ となる

また $\displaystyle M = \lim_{\theta\to+0}\theta\, e^{\left(-\frac{1}{2}\log 2 - \log|\sin\theta|\right)}$

$$= \lim_{\theta\to+0}\theta\cdot e^{\log 2^{-\frac{1}{2}}}\cdot e^{\log|\sin\theta|^{-1}}$$

$$= \lim_{\theta\to+0}\theta\cdot\left(2^{-\frac{1}{2}}\cdot\frac{1}{|\sin\theta|}\right)$$

$$= \lim_{\theta\to+0}\left(\frac{1}{\sqrt{2}}\cdot\frac{\theta}{\sin\theta}\right) = \frac{1}{\sqrt{2}}\times 1 = \boxed{\frac{\sqrt{2}}{2}}$$ となる