

$$(i) \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = - \int \frac{(\cos x)'}{\cos x} \, dx = \boxed{-\log |\cos x| + C}$$

$$\begin{aligned} (ii) \quad I(\theta) &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}-\theta} \tan x \, dx = \left[-\log |\cos x| \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}-\theta} \\ &= -\log \left| \cos \left(\frac{\pi}{2} - \theta \right) \right| + \log \left| \cos \frac{\pi}{4} \right| \\ &= -\log |\sin \theta| + \log \left(\frac{1}{\sqrt{2}} \right) \\ &= \log 2^{-\frac{1}{2}} - \log |\sin \theta| \\ &= -\frac{1}{2} \log 2 - \log |\sin \theta| \end{aligned}$$

$$\begin{aligned} \therefore L &= \lim_{\theta \rightarrow +0} (I(\theta) - I(2\theta)) \\ &= \lim_{\theta \rightarrow +0} \left\{ \left(-\frac{1}{2} \log 2 - \log |\sin \theta| \right) - \left(-\frac{1}{2} \log 2 - \log |\sin 2\theta| \right) \right\} \\ &= \lim_{\theta \rightarrow +0} (\log |\sin 2\theta| - \log |\sin \theta|) \\ &= \lim_{\theta \rightarrow +0} \log \left| \frac{2 \sin \theta \cos \theta}{\sin \theta} \right| = \lim_{\theta \rightarrow +0} \log |2 \cos \theta| = \boxed{\log 2} \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \therefore M &= \lim_{\theta \rightarrow +0} \theta e^{(-\frac{1}{2} \log 2 - \log |\sin \theta|)} \\ &= \lim_{\theta \rightarrow +0} \theta \cdot e^{\log 2^{-\frac{1}{2}}} \cdot e^{\log |\sin \theta|^{-1}} \\ &= \lim_{\theta \rightarrow +0} \theta \cdot \left(2^{-\frac{1}{2}} \cdot \frac{1}{|\sin \theta|} \right) \\ &= \lim_{\theta \rightarrow +0} \left(\frac{1}{\sqrt{2}} \cdot \frac{\theta}{\sin \theta} \right) = \frac{1}{\sqrt{2}} \times 1 = \boxed{\frac{\sqrt{2}}{2}} \quad \text{Ans} \end{aligned}$$