(1) $\log x = t$ とおくと $\dfrac{dt}{dx} = \dfrac{1}{x}$ より

$$dx = x\,dt = e^t\,dt$$

x	$1 \cdots\cdots e$
t	$0 \cdots\cdots 1$

よって
$$\int_1^e 5^{\log x}\,dx = \int_0^1 5^t \cdot e^t\,dt$$
$$= \int_0^1 (5e)^t\,dt$$
$$= \left[\frac{(5e)^t}{\log 5e}\right]_0^1$$
$$= \boxed{\frac{5e-1}{\log 5 + 1}}$$

(2) $x = \tan\theta$ とおくと $dx = \dfrac{d\theta}{\cos^2\theta}$,

x	$0 \cdots 1$
θ	$0 \cdots \frac{\pi}{4}$

より

$$\int_0^1 \frac{x+1}{(x^2+1)^2}\,dx = \int_0^{\frac{\pi}{4}} \frac{\tan\theta + 1}{(\tan^2\theta+1)^2} \cdot \frac{d\theta}{\cos^2\theta}$$

$\left(\tan^2\theta + 1 = \dfrac{1}{\cos^2\theta}\right)$

$$= \int_0^{\frac{\pi}{4}} \frac{\tan\theta+1}{\frac{1}{\cos^4\theta}} \cdot \frac{d\theta}{\cos^2\theta}$$
$$= \int_0^{\frac{\pi}{4}} \cos^2\theta\left(\frac{\sin\theta}{\cos\theta} + 1\right)d\theta$$
$$= \int_0^{\frac{\pi}{4}} (\cos\theta\sin\theta + \cos^2\theta)\,d\theta$$
$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\sin 2\theta + \frac{1+\cos 2\theta}{2}\right)d\theta$$
$$= \left[-\frac{1}{4}\cos 2\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}$$
$$= \frac{\pi}{8} + \frac{1}{4} + \frac{1}{4}$$
$$= \boxed{\frac{\pi+4}{8}}$$