

$$(1) \sin\left(x + \frac{\pi}{4}\right) = \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$$

$$= \boxed{\frac{\sqrt{2}}{2} (\sin x + \cos x)}$$

$$(2) f(x) = \sin^3 x \quad \text{or}$$

$$f'(x) = 3 \sin^2 x \cos x$$

$$(3) I = \int_0^{\frac{\pi}{6}} e^{3x} \sin^2 x \sin\left(x + \frac{\pi}{4}\right) dx \quad \text{to find}$$

$$I = \int_0^{\frac{\pi}{6}} \frac{\sqrt{2}}{2} e^{3x} \sin^2 x (\sin x + \cos x) dx$$

$$= \frac{\sqrt{2}}{2} \int_0^{\frac{\pi}{6}} (e^{3x} \sin^3 x + e^{3x} \sin^2 x \cos x) dx$$

$$= \frac{\sqrt{2}}{2} \int_0^{\frac{\pi}{6}} \frac{1}{3} \left\{ (e^{3x})' \sin^3 x + e^{3x} (\sin^3 x)' \right\} dx$$

$$= \frac{\sqrt{2}}{6} \left[e^{3x} \sin^3 x \right]_0^{\frac{\pi}{6}}$$

$$= \frac{\sqrt{2}}{6} \times e^{\frac{\pi}{2}} \sin^3 \frac{\pi}{6}$$

$$= \boxed{\frac{\sqrt{2}}{48} e^{\frac{\pi}{2}}}$$