

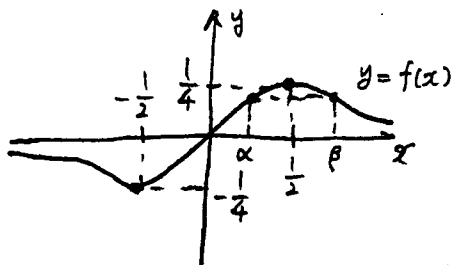
$$(1) f(x) = \frac{x}{1+4x^2} \text{ であり}$$

$$\begin{aligned} f'(x) &= \frac{1 \times (1+4x^2) - x \times 8x}{(1+4x^2)^2} \\ &= \frac{1-4x^2}{(1+4x^2)^2} \\ &= \frac{-4(x-\frac{1}{2})(x+\frac{1}{2})}{(1+4x^2)^2} \end{aligned}$$

よって増減表は以下のようになる?

x	...	$-\frac{1}{2}$	...	$\frac{1}{2}$	...
f'(x)	-	0	+	0	-
f(x)		$\rightarrow$ 極小		$\rightarrow$ 極大	

$$\begin{aligned} x = -\frac{1}{2} \text{ で極小値 } & \frac{-\frac{1}{2}}{1+4 \times (-\frac{1}{2})^2} = -\frac{1}{4} \\ x = \frac{1}{2} \text{ で極大値 } & \frac{\frac{1}{2}}{1+4 \times (\frac{1}{2})^2} = \frac{1}{4} \text{ となる} \end{aligned}$$



$$\begin{aligned} (2) \int f(x) dx &= \int \frac{x}{4x^2+1} dx \\ &= \int \frac{1}{8} \times \frac{8x}{4x^2+1} dx \\ &= \boxed{\frac{1}{8} \log(4x^2+1) + C \quad (C \text{ は定数})} \end{aligned}$$

$$(3) f(\alpha) = f(\beta) \text{ であり } \frac{\alpha}{1+4\alpha^2} = \frac{\beta}{1+4\beta^2} \text{ であるから}$$

$$\alpha(1+4\beta^2) = \beta(1+4\alpha^2) \text{ であり}$$

$$\alpha + 4\alpha\beta^2 - \beta - 4\alpha^2\beta = 0$$

$$(\alpha - \beta) - 4\alpha\beta(\alpha - \beta) = 0$$

$$(\alpha - \beta)(1 - 4\alpha\beta) = 0$$

$$\therefore \alpha - \beta \neq 0 \text{ であり } 1 - 4\alpha\beta = 0 \text{ であるから } \boxed{\alpha\beta = \frac{1}{4}}$$

$$\begin{aligned} (4) \int_{\alpha}^{\beta} f(x) dx &= \frac{1}{8} [\log(4x^2+1)]_{\alpha}^{\beta} \\ &= \frac{1}{8} \{ \log(4\beta^2+1) - \log(4\alpha^2+1) \} \\ &= \frac{1}{8} \{ \log(4\beta^2+1) - \log(4 \times (\frac{1}{4\beta})^2 + 1) \} \\ &= \frac{1}{8} \{ \log(4\beta^2+1) - \log(\frac{1}{\beta^2} + 1) \} \\ &= \frac{1}{8} \log \frac{4\beta^2+1}{\frac{1}{\beta^2}+1} \\ &= \frac{1}{8} \log \frac{4\beta^2(4\beta^2+1)}{1+4\beta^2} \\ &= \frac{1}{8} \log(4\beta^2) \\ &= \boxed{\frac{1}{4} (\log \beta + \log 2)} \text{ となる} \end{aligned}$$