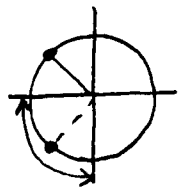
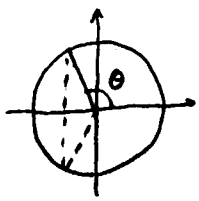
(1) $\cos x = \cos\frac{3}{4}\pi$ で $\pi \leqq x \leqq \frac{3}{2}\pi$ より

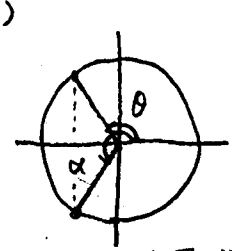
$$x = \boxed{\frac{5}{4}\pi}$$

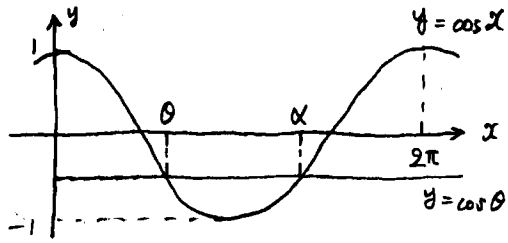


(2) 

$\frac{\pi}{2} \leqq \theta < \pi$ より $\cos x = \cos\theta$ の $0 \leqq x \leqq \frac{3}{2}\pi$ における解は

$$\boxed{x = \theta,\ 2\pi - \theta}$$ となる

(3) 



上図より

$$S = \int_0^{\alpha} |\cos x - \cos\theta|\, dx$$

$$= \int_0^{\theta} (\cos x - \cos\theta)\, dx + \int_{\theta}^{\alpha} (\cos\theta - \cos x)\, dx$$

$$= [\sin x - x\cos\theta]_0^{\theta} + [x\cos\theta - \sin x]_{\theta}^{\alpha}$$

$$= \sin\theta - \theta\cos\theta + \alpha\cos\theta - \sin\alpha - \theta\cos\theta + \sin\theta$$

$$= 2\sin\theta - 2\theta\cos\theta + (2\pi - \theta)\cos\theta - \sin(2\pi - \theta)$$

$$= 2\sin\theta - 2\theta\cos\theta + 2\pi\cos\theta - \theta\cos\theta + \sin\theta$$

$$= \boxed{3\sin\theta - 3\theta\cos\theta + 2\pi\cos\theta}$$

(4) $\frac{dS}{d\theta} = 3\cos\theta - 3\cos\theta + 3\theta\sin\theta - 2\pi\sin\theta$

$= (3\theta - 2\pi)\sin\theta$ より

$\frac{\pi}{2} \leqq \theta < \pi$ で増減表をかくと下のようになる

θ	$\frac{\pi}{2}$	…	$\frac{2\pi}{3}$	…	π
$\frac{dS}{d\theta}$		−		+	
S		↘	最小	↗	

よって $\boxed{\theta = \frac{2\pi}{3}}$ で S は最小となり

$$S = 3\sin\frac{2}{3}\pi - 3\cdot\frac{2}{3}\pi\cos\frac{2}{3}\pi + 2\pi\cos\frac{2}{3}\pi$$

$$= 3\cdot\frac{\sqrt{3}}{2} - 2\pi\times\left(-\frac{1}{2}\right) + 2\pi\times\left(-\frac{1}{2}\right)$$

$$= \boxed{\frac{3\sqrt{3}}{2}}$$ となる