$$I = \int_0^{\frac{1}{2}} (x+1)\sqrt{1-2x^2}\, dx$$ とする.

$\sqrt{2}x = \sin\theta$ とおくと $\sqrt{2}\, dx = \cos\theta\, d\theta$,

x	$0 \cdots \frac{1}{2}$
θ	$0 \cdots \frac{\pi}{4}$

より

$$I = \int_0^{\frac{\pi}{4}} \left(\frac{1}{\sqrt{2}}\sin\theta + 1\right)\sqrt{1-\sin^2\theta} \cdot \frac{1}{\sqrt{2}}\cos\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\sin\theta\cos^2\theta + \frac{1}{\sqrt{2}}\cos^2\theta\right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\sin\theta\cos^2\theta + \frac{1}{\sqrt{2}} \times \frac{\cos 2\theta + 1}{2}\right) d\theta$$

$$= \left[-\frac{1}{6}\cos^3\theta + \frac{1}{2\sqrt{2}} \times \frac{1}{2}\sin 2\theta + \frac{1}{2\sqrt{2}}\theta \right]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{6} \times \frac{1}{2\sqrt{2}} + \frac{1}{6} \times 1 + \frac{1}{4\sqrt{2}} \times 1 + \frac{1}{2\sqrt{2}} \times \frac{\pi}{4}$$

$$= -\frac{\sqrt{2}}{24} + \frac{4}{24} + \frac{3\sqrt{2}}{24} + \frac{\sqrt{2}\pi}{16}$$

$$= \frac{\sqrt{2}}{12} + \frac{2}{12} + \frac{\sqrt{2}\pi}{16}$$

$$= \boxed{\frac{4\sqrt{2} + 8 + 3\sqrt{2}\pi}{48}}$$