(1) $$\frac{d}{dx}\log\left(2x+\sqrt{4x^2+1}\right) = \frac{\left(2x+\sqrt{4x^2+1}\right)'}{2x+\sqrt{4x^2+1}} = \frac{2+\dfrac{\cancel{8x}^{\,4}}{\cancel{2}_{\,1}\sqrt{4x^2+1}}}{2x+\sqrt{4x^2+1}}$$

$$= \frac{1}{2x+\sqrt{4x^2+1}} \times \frac{2\sqrt{4x^2+1}+4x}{\sqrt{4x^2+1}}$$

$$= \frac{2\cancel{\left(2x+\sqrt{4x^2+1}\right)}}{\cancel{\left(2x+\sqrt{4x^2+1}\right)}\sqrt{4x^2+1}}$$

$$= \boxed{\frac{\boxed{2}}{\sqrt{4x^2+1}}} \ \text{ア} \quad \text{——①}$$

$$\frac{d}{dx}\left(x\sqrt{4x^2+1}\right) = 1\times\sqrt{4x^2+1} + x\times\frac{\cancel{8x}^{\,4}}{\cancel{2}_{\,1}\sqrt{4x^2+1}}$$

$$= \frac{(4x^2+1)+4x^2}{\sqrt{4x^2+1}}$$

$$= \frac{2(4x^2+1)-1}{\sqrt{4x^2+1}} = \boxed{2\sqrt{4x^2+1} - \frac{\boxed{1}}{\sqrt{4x^2+1}}} \ \text{イ} \quad \text{となる}$$

└②

よって ②より $\dfrac{1}{2}\cdot\dfrac{d}{dx}\left(x\sqrt{4x^2+1}\right) = \sqrt{4x^2+1} - \dfrac{1}{4}\times\dfrac{2}{\sqrt{4x^2+1}}$ であり

①より $\dfrac{1}{2}\cdot\dfrac{d}{dx}\left(x\sqrt{4x^2+1}\right) = \sqrt{4x^2+1} - \dfrac{1}{4}\cdot\dfrac{d}{dx}\log\left(2x+\sqrt{4x^2+1}\right)$ であるから

$\sqrt{4x^2+1} = \dfrac{1}{2}\cdot\dfrac{d}{dx}\left(x\sqrt{4x^2+1}\right) + \dfrac{1}{4}\cdot\dfrac{d}{dx}\log\left(2x+\sqrt{4x^2+1}\right)$ となる

ゆえに $$\int_0^1 \sqrt{4x^2+1}\,dx = \frac{1}{2}\left[x\sqrt{4x^2+1}\right]_0^1 + \frac{1}{4}\left[\log\left(2x+\sqrt{4x^2+1}\right)\right]_0^1$$

$$= \frac{1}{2}\times1\times\sqrt{5} + \frac{1}{4}\log\left(2+\sqrt{5}\right) - \frac{1}{4}\log 1$$

$$= \boxed{\frac{\sqrt{5}}{2} + \frac{1}{4}\log\left(2+\sqrt{5}\right)}$$