$$\tan\frac{5\pi}{12} = \tan\left(\frac{2}{12}\pi + \frac{3}{12}\pi\right)$$

$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$$

$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \times 1} = \frac{1+\sqrt{3}}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{2} = \frac{4+2\sqrt{3}}{2} = \overset{(7)}{\boxed{2+\sqrt{3}}}$$

である

また $I = \int_0^{2(2+\sqrt{3})} \frac{16}{(x^2+4)^2}\,dx$ において

$x = 2\tan\theta$ とおくと $dx = \frac{2\,d\theta}{\cos^2\theta}$ ，

x	$0 \cdots 2(2+\sqrt{3})$
θ	$0 \cdots \frac{5\pi}{12}$

より

$$I = \int_0^{\frac{5\pi}{12}} \frac{16}{(4\tan^2\theta + 4)^2} \cdot \frac{2\,d\theta}{\cos^2\theta}$$

$$= \int_0^{\frac{5\pi}{12}} \frac{1}{(\tan^2\theta + 1)^2} \cdot \frac{2\,d\theta}{\cos^2\theta}$$

$$= \int_0^{\frac{5}{12}\pi} \frac{1}{\frac{1}{\cos^4\theta}} \cdot \frac{2\,d\theta}{\cos^2\theta}$$

$$= \int_0^{\frac{5\pi}{12}} 2\cos^2\theta\,d\theta$$

$$= \int_0^{\frac{5\pi}{12}} (1+\cos 2\theta)\,d\theta$$

$$= \left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{5\pi}{12}}$$

$$= \frac{5}{12}\pi + \frac{1}{2}\times\sin\frac{5}{6}\pi$$

$$= \frac{5}{12}\pi + \frac{1}{2}\times\frac{1}{2}$$

$$= \boxed{\frac{5\pi+3}{12}}$$