

$$\frac{x-1}{x^3+1} = \frac{\alpha x + \beta}{x^2-x+1} + \frac{r}{x+1} \quad \text{57}$$

$$\frac{x-1}{x^3+1} = \frac{(\alpha x + \beta)(x+1) + r(x^2-x+1)}{x^3+1}$$

$$\frac{x-1}{x^3+1} = \frac{(\alpha+r)x^2 + (\alpha+\beta-r)x + (\beta+r)}{x^3+1}$$

$$\text{57} \quad \left\{ \begin{array}{l} \alpha + r = 0 \quad \text{--- ①} \\ \alpha + \beta - r = 1 \quad \text{--- ②} \\ \beta + r = -1 \quad \text{--- ③} \end{array} \right. \quad \text{77 87 127.}$$

$$\text{① 57} \quad \alpha = -r$$

$$\text{③ 57} \quad \beta = -r - 1 \quad \text{77 87 ② 12}$$

$$-r - r - 1 - r = 1 \quad \text{57}$$

$$\text{77 12} \quad \begin{array}{l} -3r = 2 \\ r = -\frac{2}{3} \end{array} \quad \text{57} \quad \alpha = \frac{2}{3}, \quad \beta = \frac{2}{3} - 1 = -\frac{1}{3} \quad \text{77 87 12}$$

$$\int \frac{x-1}{x^3+1} dx = \int \frac{1}{3} \cdot \frac{2x-1}{x^2-x+1} dx - \frac{2}{3} \int \frac{1}{x+1} dx$$

$$= \frac{1}{3} \log |x^2-x+1| - \frac{2}{3} \log |x+1| + C \quad \leftarrow \left(x^2-x+1 = \left(x-\frac{1}{2}\right)^2 + \frac{3}{4} > 0 \right)$$

$$= \frac{1}{3} \log (x^2-x+1) - \frac{1}{3} \log (x+1)^2 + C$$

$$= \frac{1}{3} \log \frac{x^2-x+1}{(x+1)^2} + C \quad (C \text{ は定数})$$

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