

$$\begin{aligned}
 \int \frac{e^{-2x}}{1+e^{-x}} dx &= \int \frac{1}{e^{2x}+e^x} dx \\
 &= \int \frac{1}{e^x(e^x+1)} dx \\
 &= \int \left(\frac{1}{e^x} - \frac{1}{e^x+1} \right) dx
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{let } t = e^x \quad \text{and} \quad dt = e^x \cdot dx \quad \text{so} \\
 dx = \frac{dt}{t}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ (5式)} &= \int \left(\frac{1}{t} - \frac{1}{t+1} \right) \frac{dt}{t} \\
 &= \int \left(\frac{1}{t^2} - \frac{1}{t(t+1)} \right) dt \\
 &= \int \left\{ \frac{1}{t^2} - \left(\frac{1}{t} - \frac{1}{t+1} \right) \right\} dt \\
 &= \int \left(\frac{1}{t^2} - \frac{1}{t} + \frac{1}{t+1} \right) dt \\
 &= -\frac{1}{t} - \log|t| + \log|t+1| + C \\
 &= -e^{-x} - \log e^x + \log(e^x+1) + C \\
 &= \boxed{-e^{-x} - x + \log(e^x+1) + C} \\
 &\quad (C \text{ is a constant})
 \end{aligned}$$