$$\int \frac{e^{-2x}}{1+e^{-x}}dx = \int \frac{1}{e^{2x}+e^{x}}dx$$

$$= \int \frac{1}{e^{x}(e^{x}+1)}dx$$

$$= \int \left(\frac{1}{e^{x}} - \frac{1}{e^{x}+1}\right)dx$$

ここで $t = e^{x}$ とおくと $dt = e^{x} \cdot dx$ より

$$dx = \frac{dt}{t}$$

よって (与式) $= \int \left(\frac{1}{t} - \frac{1}{t+1}\right)\frac{dt}{t}$

$$= \int \left(\frac{1}{t^{2}} - \frac{1}{t(t+1)}\right)dt$$

$$= \int \left\{\frac{1}{t^{2}} - \left(\frac{1}{t} - \frac{1}{t+1}\right)\right\}dt$$

$$= \int \left(\frac{1}{t^{2}} - \frac{1}{t} + \frac{1}{t+1}\right)dt$$

$$= -\frac{1}{t} - \log|t| + \log|t+1| + C$$

$$= -e^{-x} - \log e^{x} + \log(e^{x}+1) + C$$

$$= \boxed{-e^{-x} - x + \log(e^{x}+1) + C}$$

(Cは定数)