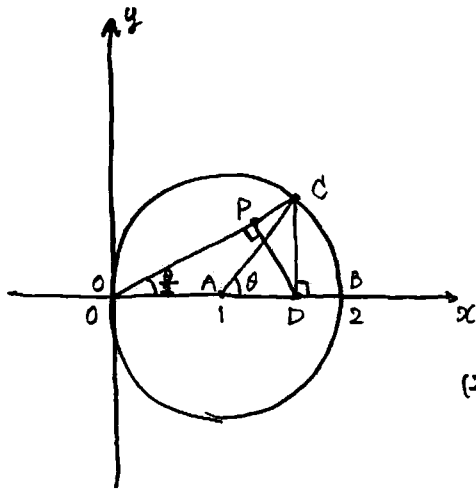




$$(1) C \text{ は } (1+\cos\theta, \sin\theta)$$

$$D \text{ は } (1+\cos\theta, 0) \text{ であり}$$

$$\angle POD = \frac{\theta}{2} \text{ である}$$

$$P \text{ は } \left((1+\cos\theta) \cos \frac{\theta}{2}, (1+\cos\theta) \cos \frac{\theta}{2} \sin \frac{\theta}{2} \right)$$

$$(2) \vec{OP} = (x, y) \text{ とおくと}$$

$$x = (1+\cos\theta) \cdot \cos \frac{\theta}{2} = \frac{(1+\cos\theta)^2}{2}$$

$$y = (1+\cos\theta) \times \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \frac{(1+\cos\theta) \sin \theta}{2} \quad \text{すなわち}$$

$$r^2 = |\vec{OP}|^2 = \frac{(1+\cos\theta)^4}{4} + \frac{(1+\cos\theta)^2 \sin^2 \theta}{4}$$

$$= \frac{1}{4} (1+\cos\theta)^2 \{ (1+\cos\theta)^2 + \sin^2 \theta \}$$

$$= \frac{1}{4} (1+\cos\theta)^2 (1+2\cos\theta+1)$$

$$= \frac{1}{4} (1+\cos\theta)^2 \cdot (2+2\cos\theta)$$

$$= \frac{1}{4} (1+\cos\theta)^2 \cdot 4 \times \frac{1+\cos\theta}{2}$$

$$= (1+\cos\theta)^2 \cdot \cos \frac{\theta}{2}$$

$$\text{すなわち } r = (1+\cos\theta) \cdot \cos \frac{\theta}{2}$$

$$\begin{aligned} \cos(\alpha+\beta) &= \cos\alpha \cos\beta - \sin\alpha \sin\beta \\ + \cos(\alpha-\beta) &= \cos\alpha \cos\beta + \sin\alpha \sin\beta \\ \hline \cos(\alpha+\beta) + \cos(\alpha-\beta) &= 2\cos\alpha \cos\beta \end{aligned}$$

$$\begin{aligned} \text{また } \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} r d\theta &= \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (1+\cos\theta) \cdot \cos \frac{\theta}{2} d\theta \\ &= \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(\cos \frac{\theta}{2} + \cos\theta \cdot \cos \frac{\theta}{2} \right) d\theta \\ &= \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(\cos \frac{\theta}{2} + \frac{1}{2} \cos \frac{3}{2}\theta + \frac{1}{2} \cos \frac{1}{2}\theta \right) d\theta \\ &= \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(\frac{3}{2} \cos \frac{\theta}{2} + \frac{1}{2} \cos \frac{3}{2}\theta \right) d\theta \\ &= \left[\frac{3}{2} \cdot 2 \cdot \sin \frac{\theta}{2} + \frac{1}{2} \cdot \frac{2}{3} \sin \frac{3}{2}\theta \right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \\ &= 3 \sin \frac{\pi}{3} + \frac{1}{3} \sin \pi - 3 \sin \frac{\pi}{6} - \frac{1}{3} \sin \frac{\pi}{2} \\ &= \frac{3\sqrt{3}}{2} + 0 - \frac{3}{2} - \frac{1}{3} \\ &= \frac{-11 + 9\sqrt{3}}{6} \end{aligned}$$