(1) $\displaystyle\int e^{-x}\sin 2x\,dx = I$ とおくと

$$I = \int (-e^{-x})'\sin 2x\,dx$$

$$= -e^{-x}\sin 2x + \int e^{-x}\cdot 2\cos 2x\,dx + C_1 \quad (C_1\text{は定数})$$

$$= -e^{-x}\sin 2x + \int (-e^{-x})'\cdot 2\cos 2x\,dx + C_1$$

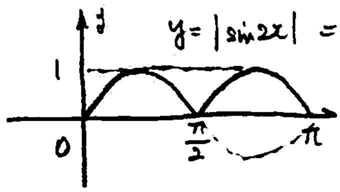
$$= -e^{-x}\sin 2x - e^{-x}\cdot 2\cos 2x + \int e^{-x}\cdot(-4)\cdot\sin 2x\,dx + C_2 \quad (C_2\text{は定数})$$

$$= -e^{-x}(\sin 2x + 2\cos 2x) - 4I + C_2$$

よって　$5I = -e^{-x}(\sin 2x + 2\cos 2x) + C_2$ より

$$I = \boxed{-\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x) + C \quad (C\text{は定数})}$$

(2) $\displaystyle\int_0^{\pi} e^{-x}|\sin 2x|\,dx$



$$y = |\sin 2x| = \begin{cases} \sin 2x & (0 \leqq x < \frac{\pi}{2}) \\ -\sin 2x & (\frac{\pi}{2} \leqq x \leqq \pi) \end{cases}$$

$$= \int_0^{\frac{\pi}{2}} e^{-x}\sin 2x\,dx - \int_{\frac{\pi}{2}}^{\pi} e^{-x}\sin 2x\,dx$$

$$= \left[-\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x)\right]_0^{\frac{\pi}{2}} - \left[-\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x)\right]_{\frac{\pi}{2}}^{\pi}$$

$$= -\frac{1}{5}e^{-\frac{\pi}{2}}\times(-2) + \frac{1}{5}e^{0}\times 2 + \frac{1}{5}e^{-\pi}\times 2 - \frac{1}{5}e^{-\frac{\pi}{2}}\times(-2)$$

$$= \frac{4}{5}e^{-\frac{\pi}{2}} + \frac{2}{5}e^{-\pi} + \frac{2}{5}$$

$$= \boxed{\frac{2}{5}\left(2e^{-\frac{\pi}{2}} + e^{-\pi} + 1\right)}$$