

$$(1) \quad 1 + \tan^2\left(\frac{x}{2}\right) = \frac{1}{\cos^2\left(\frac{x}{2}\right)} = \frac{2}{1 + \cos x} \quad \text{よって}$$

$$1 + t^2 = \frac{2}{1 + \cos x}$$

$$\text{よって } 1 + \cos x = \frac{2}{1 + t^2} \quad \text{よって}$$

$$\cos x = \frac{2}{1 + t^2} - \frac{1 + t^2}{1 + t^2}$$

$$\boxed{\cos x = \frac{1 - t^2}{1 + t^2}}$$

$$\text{また } \tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} = \frac{2t}{1 - t^2} \quad \text{よって}$$

$$\boxed{\sin x} = \tan x \cos x = \frac{2t}{1 - t^2} \times \frac{1 - t^2}{1 + t^2} = \boxed{\frac{2t}{1 + t^2}}$$

$$(2) \quad t = \tan \frac{x}{2} \quad \text{よって}$$

$$\frac{dt}{dx} = \frac{\frac{1}{2}}{\cos^2\left(\frac{x}{2}\right)} = \frac{1}{2} \left\{ 1 + \tan^2\left(\frac{x}{2}\right) \right\} = \frac{1 + t^2}{2}$$

$$\text{よって } \boxed{\frac{dx}{dt}} = \frac{1}{\frac{dt}{dx}} = \boxed{\frac{2}{1 + t^2}}$$

$$\begin{aligned} (3) \quad & \int \frac{5}{3 \sin x + 4 \cos x} dx \\ &= \int \frac{5}{3 \times \frac{2t}{1+t^2} + 4 \times \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2} dt \\ &= \int \frac{5}{6t + 4(1-t^2)} \times 2 dt \\ &= \int \frac{5}{3t + 2 - 2t^2} dt \\ &= \int \frac{-5}{(t-2)(2t+1)} dt \\ &= -\frac{5}{2} \int \frac{1}{(t-2)(t+\frac{1}{2})} dt \\ &= -\frac{5}{2} \times \frac{2}{5} \int \left(\frac{1}{t-2} - \frac{1}{t+\frac{1}{2}} \right) dt \\ &= \int \left(\frac{1}{t+\frac{1}{2}} - \frac{1}{t-2} \right) dt \end{aligned}$$

$$= \log \left| t + \frac{1}{2} \right| - \log |t - 2| + C$$

$$= \log \left| \frac{t + \frac{1}{2}}{t - 2} \right| + C$$

$$= \boxed{\log \left| \frac{\tan \frac{x}{2} + \frac{1}{2}}{\tan \frac{x}{2} - 2} \right| + C}$$

(Cは定数)