

$$\int_0^2 \frac{2x+1}{\sqrt{x^2+4}} dx$$

$$x = 2 \tan \theta \quad 0 < \theta < \frac{\pi}{4} \quad dx = 2 \times \frac{d\theta}{\cos^2 \theta} \quad 2''$$

$$\begin{array}{c|c} x & 0 \dots 2 \\ \theta & 0 \dots \frac{\pi}{4} \end{array} \quad 2''$$

$$\begin{aligned} (5'') &= \int_0^{\frac{\pi}{4}} \frac{2 \times 2 \tan \theta + 1}{\sqrt{4 \tan^2 \theta + 4}} \times \frac{2 d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{4}} \frac{4 \tan \theta + 1}{\sqrt{4(1 + \tan^2 \theta)}} \times \frac{2 d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{4}} \frac{4 \tan \theta + 1}{2 \times \frac{1}{\cos \theta}} \times \frac{2 d\theta}{\cos^2 \theta} \\ &= \int_0^{\frac{\pi}{4}} \frac{4 \tan \theta + 1}{\cos \theta} d\theta \\ &= \int_0^{\frac{\pi}{4}} \left(\frac{4 \sin \theta}{\cos^2 \theta} + \frac{1}{\cos \theta} \right) d\theta \\ &= \int_0^{\frac{\pi}{4}} \left(\frac{4 \sin \theta}{\cos^2 \theta} + \frac{\cos \theta}{\cos^2 \theta} \right) d\theta \\ &= \int_0^{\frac{\pi}{4}} \left(4 \times \frac{\sin \theta}{\cos^2 \theta} + \frac{\cos \theta}{1 - \sin^2 \theta} \right) d\theta \\ &= \int_0^{\frac{\pi}{4}} \left\{ 4 \times \frac{\sin \theta}{\cos^2 \theta} + \frac{1}{2} \left(\frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} \right) \right\} d\theta \\ &= \left[4 \times \frac{1}{\cos \theta} - \frac{1}{2} \log |1 - \sin \theta| + \frac{1}{2} \log |1 + \sin \theta| \right]_0^{\frac{\pi}{4}} \\ &= \left[\frac{4}{\cos \theta} + \frac{1}{2} \log \left| \frac{1 + \sin \theta}{1 - \sin \theta} \right| \right]_0^{\frac{\pi}{4}} \\ &= \frac{4}{\frac{1}{\sqrt{2}}} - \frac{4}{1} + \frac{1}{2} \log \left| \frac{1 + \frac{\sqrt{2}}{2}}{1 - \frac{\sqrt{2}}{2}} \right| - \frac{1}{2} \log 1 \\ &= 4\sqrt{2} - 4 + \frac{1}{2} \log \left| \frac{2 + \sqrt{2}}{2 - \sqrt{2}} \right| \\ &= 4\sqrt{2} - 4 + \frac{1}{2} \log \left| \frac{(2 + \sqrt{2})^2}{4 - 2} \right| \\ &= 4\sqrt{2} - 4 + \frac{1}{2} \log \frac{(2 + \sqrt{2})^2}{2} \\ &= 4\sqrt{2} - 4 + \log \frac{2 + \sqrt{2}}{\sqrt{2}} = \boxed{4\sqrt{2} - 4 + \log(\sqrt{2} + 1)} \end{aligned}$$