$\int_0^{\frac{\log 3}{2}} \frac{e^x+1}{e^{2x}+1}\,dx$ において $t=e^x$ とおくと $dt=e^x\,dx$

x	$0 \cdots \frac{\log 3}{2}$
t	$1 \cdots \sqrt{3}$

より

$$(与式) = \int_1^{\sqrt{3}} \frac{t+1}{t^2+1} \times \frac{1}{t}\,dt$$

$$= \int_1^{\sqrt{3}} \left(\frac{1}{t^2+1} + \frac{1}{t} \times \frac{1}{t^2+1} \right) dt$$

$$= \int_1^{\sqrt{3}} \left(\frac{1}{t^2+1} + \frac{1}{t} - \frac{t}{t^2+1} \right) dt$$

$$= \int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt + \left[\log|t| - \frac{1}{2}\log(t^2+1) \right]_1^{\sqrt{3}}$$

$$= \int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt + \log\sqrt{3} - \frac{1}{2}\log 4 + \frac{1}{2}\log 2$$

$$= \int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt + \frac{1}{2}(\log 3 - \log 4 + \log 2)$$

$$= \int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt + \frac{1}{2}\log\frac{3\times 2}{4}$$

$$= \int_1^{\sqrt{3}} \frac{1}{t^2+1}\,dt + \frac{1}{2}\log\frac{3}{2}$$

ここで $t=\tan\theta$ とおくと $dt = \frac{1}{\cos^2\theta}\,d\theta$,

t	$1 \cdots \sqrt{3}$
θ	$\frac{\pi}{4} \cdots \frac{\pi}{3}$

より

$$(与式) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{1+\tan^2\theta} \times \frac{d\theta}{\cos^2\theta} + \frac{1}{2}\log\frac{3}{2}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2\theta \times \frac{1}{\cos^2\theta}\,d\theta + \frac{1}{2}\log\frac{3}{2}$$

$$= \left[\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} + \frac{1}{2}\log\frac{3}{2}$$

$$= \left(\frac{\pi}{3} - \frac{\pi}{4}\right) + \frac{1}{2}\log\frac{3}{2}$$

$$= \boxed{\frac{\pi}{12} + \frac{1}{2}\log\frac{3}{2}}$$