

$$\begin{aligned}
 (1) \quad & \frac{d}{dx} (x \sin x + \cos x) \\
 &= 1 \times \sin x + x \cos x - \sin x \\
 &= x \cos x
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \int_{2n\pi}^{2(n+1)\pi} \frac{x \sin x + \cos x}{x^2} dx \\
 &= \int_{2n\pi}^{2(n+1)\pi} \left(-\frac{1}{x}\right)' (x \sin x + \cos x) dx \\
 &= \left[-\frac{x \sin x + \cos x}{x} \right]_{2n\pi}^{2(n+1)\pi} + \int_{2n\pi}^{2(n+1)\pi} \frac{1}{x} \times x \cos x dx \\
 &= -\frac{1}{2(n+1)\pi} + \frac{1}{2n\pi} + [\sin x]_{2n\pi}^{2(n+1)\pi} \\
 &= \boxed{\frac{1}{2\pi} \left(\frac{1}{n} - \frac{1}{n+1} \right)}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{2\pi} \left(\frac{1}{n} - \frac{1}{n+1} \right) \\
 &= \frac{1}{2\pi} \times \lim_{n \rightarrow \infty} \left\{ \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1} \right) \right\} \\
 &= \frac{1}{2\pi} \times \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) \\
 &= \boxed{\frac{1}{2\pi}}
 \end{aligned}$$