(1) $\frac{d}{dx}(x\sin x + \cos x)$

$= 1\times\sin x + x\cos x - \sin x$

$= x\cos x$

(2) $\int_{2n\pi}^{2(n+1)\pi} \frac{x\sin x + \cos x}{x^2}\,dx$

$= \int_{2n\pi}^{2(n+1)\pi} \left(-\frac{1}{x}\right)'(x\sin x + \cos x)\,dx$

$= \left[-\frac{x\sin x + \cos x}{x}\right]_{2n\pi}^{2(n+1)\pi} + \int_{2n\pi}^{2(n+1)\pi} \frac{1}{x}\times x\cos x\,dx$

$= -\frac{1}{2(n+1)\pi} + \frac{1}{2n\pi} + [\sin x]_{2n\pi}^{2(n+1)\pi}$

$= \boxed{\frac{1}{2\pi}\left(\frac{1}{n} - \frac{1}{n+1}\right)}$

(3) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{2\pi}\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \frac{1}{2\pi}\times \lim_{n\to\infty}\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right\}$

$= \frac{1}{2\pi}\times \lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)$

$= \boxed{\frac{1}{2\pi}}$