(1) $f(t)=\frac{1}{\pi}\int_0^{\pi}(t-\sin x)\,dx$ より

$$f(t)=\frac{1}{\pi}\Big[\,tx+\cos x\,\Big]_0^{\pi}$$
$$=\frac{1}{\pi}(t\pi-1-1)$$
$$=\frac{t\pi-2}{\pi}$$
$$=t-\frac{2}{\pi}$$

よって $f(t)=0$ となる t は $\boxed{t=\frac{2}{\pi}}$

(2) $g(t)=\frac{1}{\pi}\int_0^{\pi}(t-\sin x)^2\,dx$

$$=\frac{1}{\pi}\int_0^{\pi}(t^2-2t\sin x+\sin^2 x)\,dx$$
$$=\frac{1}{\pi}\int_0^{\pi}\left(t^2-2t\sin x+\frac{1-\cos 2x}{2}\right)dx$$
$$=\frac{1}{\pi}\Big[\,t^2x+2t\cos x+\frac{1}{2}x-\frac{1}{4}\sin 2x\,\Big]_0^{\pi}$$
$$=\frac{1}{\pi}\left\{t^2\pi+2t\times(-1)+\frac{\pi}{2}-2t\times 1\right\}$$
$$=\frac{1}{\pi}\left(t^2\pi-4t+\frac{\pi}{2}\right)$$

よって $g'(t)=\frac{1}{\pi}(2t\pi-4)=2\left(t-\frac{2}{\pi}\right)$ だから

t	$\cdots$	$\frac{2}{\pi}$	$\cdots$
$g'(t)$	$-$	0	$+$
$g(t)$	$\searrow$	最小	$\nearrow$

$g\left(\frac{2}{\pi}\right)$ が最小値をとるので

$$g\left(\frac{2}{\pi}\right)=\frac{1}{\pi}\left(\frac{4}{\pi^2}\times\pi-4\times\frac{2}{\pi}+\frac{\pi}{2}\right)$$
$$=\frac{1}{\pi}\left(\frac{4}{\pi}-\frac{8}{\pi}+\frac{\pi}{2}\right)$$
$$=\frac{1}{\pi}\left(\frac{\pi}{2}-\frac{4}{\pi}\right)$$
$$=\frac{1}{2}-\frac{4}{\pi^2}$$

ゆえに $\boxed{t=\frac{2}{\pi}\text{ のとき } g(t) \text{ は最小値 } \frac{1}{2}-\frac{4}{\pi^2} \text{ をとる}}$