(1) $\boxed{I_0} = \int_0^{\frac{\pi}{2}} (\cos x)^0 dx = \int_0^{\frac{\pi}{2}} 1 \cdot dx = [x]_0^{\frac{\pi}{2}} = \boxed{\frac{\pi}{2}}$

$\boxed{I_1} = \int_0^{\frac{\pi}{2}} (\cos x)^1 dx = [\sin x]_0^{\frac{\pi}{2}} = \boxed{1}$

$\boxed{I_2} = \int_0^{\frac{\pi}{2}} (\cos x)^2 dx = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$

$= \left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}} = \boxed{\frac{\pi}{4}}$

(2) $n \geqq 2$ のとき

$I_n = \int_0^{\frac{\pi}{2}} \cos^{n-1} x \cdot \cos x \, dx$

$= \int_0^{\frac{\pi}{2}} \cos^{n-1} x \cdot (\sin x)' dx$

$= \left[\cos^{n-1} x \cdot \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (n-1)\cos^{n-2} x \cdot (-\sin x) \cdot \sin x \cdot dx$

$= 0 + \int_0^{\frac{\pi}{2}} (n-1)\cos^{n-2} x \cdot \sin^2 x \, dx$

$= (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} x \cdot (1-\cos^2 x) dx$

$= (n-1)\left(\int_0^{\frac{\pi}{2}} \cos^{n-2} x \, dx - \int_0^{\frac{\pi}{2}} \cos^n x \, dx\right)$

よって $I_n = (n-1)(I_{n-2} - I_n)$ より

$I_n = (n-1) I_{n-2} - (n-1) I_n$

$I_n \times (1+n-1) = (n-1) I_{n-2}$

よって $\boxed{I_n = \frac{n-1}{n} I_{n-2}}$