(1) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x \cos x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin 2x \, dx$

$$= \left[-\frac{1}{4}\cos 2x \right]_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{4}(\cos\pi - \cos 0)$$

$$= -\frac{1}{4}\{(-1) - 1\} = \boxed{\frac{1}{2}}$$

(2) $x = \frac{\pi}{2} - t$ とおくと $dx = -dt$ で $\begin{array}{c|ccc} x & 0 & \cdots\cdots & \frac{\pi}{2} \\ \hline t & \frac{\pi}{2} & \cdots\cdots & 0 \end{array}$ より

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx$$

$$= \int_{\frac{\pi}{2}}^{0} \frac{\sin^3\left(\frac{\pi}{2} - t\right)}{\sin\left(\frac{\pi}{2} - t\right) + \cos\left(\frac{\pi}{2} - t\right)} \cdot (-dt)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^3 t}{\cos t + \sin t} dt$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^3 t}{\sin t + \cos t} dt.$$

(3) $\displaystyle I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\sin x + \cos x} dx$ より

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cancel{(\sin x + \cos x)}(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cancel{\sin x + \cos x}} dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - \sin x \cos x) \, dx$$

$$= [x]_0^{\frac{\pi}{2}} - \frac{1}{2}$$

$$= \frac{1}{2}(\pi - 1)$$

よって $\boxed{I = \frac{1}{4}(\pi - 1)}$