No

Date . .

$$
\begin{aligned}
I_n &= \int_0^\pi \sin^n\theta \, d\theta \\
&= \int_0^\pi \sin^{n-1}\theta \, \sin\theta \, d\theta \\
&= \int_0^\pi \sin^{n-1}\theta \, (-\cos\theta)' d\theta \\
&= \left[-\cos\theta \, \sin^{n-1}\theta \right]_0^\pi + \int_0^\pi (n-1)\sin^{n-2}\theta \cos^2\theta \, d\theta \\
&= 0 + (n-1)\int_0^\pi \sin^{n-2}\theta \,(1-\sin^2\theta)\, d\theta \\
&= (n-1)\left(\int_0^\pi \sin^{n-2}\theta \, d\theta - \int_0^\pi \sin^n\theta \, d\theta \right) \\
&= (n-1)(I_{n-2} - I_n)
\end{aligned}
$$

よって　$I_n + (n-1) I_n = (n-1) I_{n-2}$ から

$$\boxed{I_n = \frac{n-1}{n} I_{n-2}}$$

No

Date

$$J_n = \int_0^1 x^n (1-x^2)^{\frac{n-1}{2}} dx$$

$x = \sin\frac{\theta}{2}$ とおくと

x	$0 \cdots 1$
θ	$0 \cdots \pi$

$dx = \frac{1}{2}\cos\frac{\theta}{2} d\theta$ より

$$\boxed{J_n} = \int_0^\pi \sin^n\frac{\theta}{2} \left(1-\sin^2\frac{\theta}{2}\right)^{\frac{n-1}{2}} \frac{1}{2}\cos\frac{\theta}{2} d\theta$$

$$= \frac{1}{2}\int_0^\pi \sin^n\frac{\theta}{2} \cos^{n-1}\frac{\theta}{2} \cos\frac{\theta}{2} d\theta$$

$$= \frac{1}{2}\int_0^\pi \sin^n\frac{\theta}{2} \cos^n\frac{\theta}{2} d\theta$$

$$= \frac{1}{2}\int_0^\pi \left(\frac{1}{2}\sin\theta\right)^n d\theta$$

$$= \frac{1}{2} \times \left(\frac{1}{2}\right)^n \times \int_0^\pi \sin^n\theta\, d\theta$$

$$= \boxed{\frac{1}{2^{n+1}} I_n}$$

(3) $J_6 = \frac{1}{2^7} I_6$

$$= \frac{1}{2^7} \times \frac{5}{\cancel{6}_2} \times \frac{\cancel{3}^1}{4} \times \frac{1}{2} \times \pi$$

$$= \boxed{\frac{5\pi}{2048}}$$