

$$\int \frac{x^3}{(x-1)^3(x-2)} dx$$

$$\frac{x^3}{(x-1)^3(x-2)} = \frac{A}{(x-1)^3} + \frac{B}{(x-1)^2} + \frac{C}{x-1} + \frac{D}{x-2} \quad \text{とおく}$$

$$x^3 = A(x-2) + B(x-1)(x-2) + C(x-1)^2(x-2) + D(x-1)^3 \quad \text{--- ①}$$

$$x=2 \text{ 代入して}$$

$$\boxed{8 = D}$$

$$x=1 \text{ 代入して}$$

$$1 = -A \rightarrow \boxed{A = -1}$$

$$x=3 \text{ 代入して}$$

$$27 = -1 \times 1 + B \times 2 + C \times 4 + 8 \times 8$$

$$27 = -1 + 2B + 4C + 64$$

$$2B + 4C = 28 - 64 = -36$$

$$\boxed{B + 2C = -18}$$

$$x=0 \text{ 代入して}$$

$$0 = -1 \times (-2) + B \times (-1) \times (-2) + C \times 1 \times (-2) + 8 \times (-1)$$

$$0 = 2 + 2B - 2C - 8$$

$$0 = 1 + B - C - 4$$

$$0 = 1 - 18 - 2C - C - 4$$

$$3C = -21$$

$$\boxed{C = -7}$$

$$\boxed{B = -18 - 2C}$$

$$= -18 - 2 \times (-7) = -18 + 14 = \boxed{-4}$$

$$\text{5.7 (与式)} = \int \left(\frac{-1}{(x-1)^3} - \frac{4}{(x-1)^2} - \frac{7}{x-1} + \frac{8}{x-2} \right) dx$$

$$= -\frac{1}{2} \times \frac{-1}{(x-1)^2} + \frac{4}{(x-1)} - 7 \log|x-1| + 8 \log|x-2| + C$$

$$= \frac{1 + 8(x-1)}{2(x-1)^2} + \log \frac{(x-2)^8}{|x-1|^7} + C$$

$$= \left[\frac{8x-7}{2(x-1)^2} + \log \frac{(x-2)^8}{|x-1|^7} + C \right] \quad (C \text{ は積分定数})$$