

$$\begin{aligned}\int_0^1 x e^x dx &= \int_0^1 x (e^x)' dx \\&= [x e^x]_0^1 - \int_0^1 e^x dx \\&= 1 \times e^1 - 0 - [e^x]_0^1 \\&= e - (e^1 - e^0) = \boxed{1}\end{aligned}$$