(1) $y' = -e^{-x}\sin x + e^{-x}\cos x$

$= -e^{-x}(\sin x - \cos x)$

$= -e^{-x}\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)$

$= -\sqrt{2}e^{-x}\sin\left(x - \frac{\pi}{4}\right)$

ゆえに増減表は

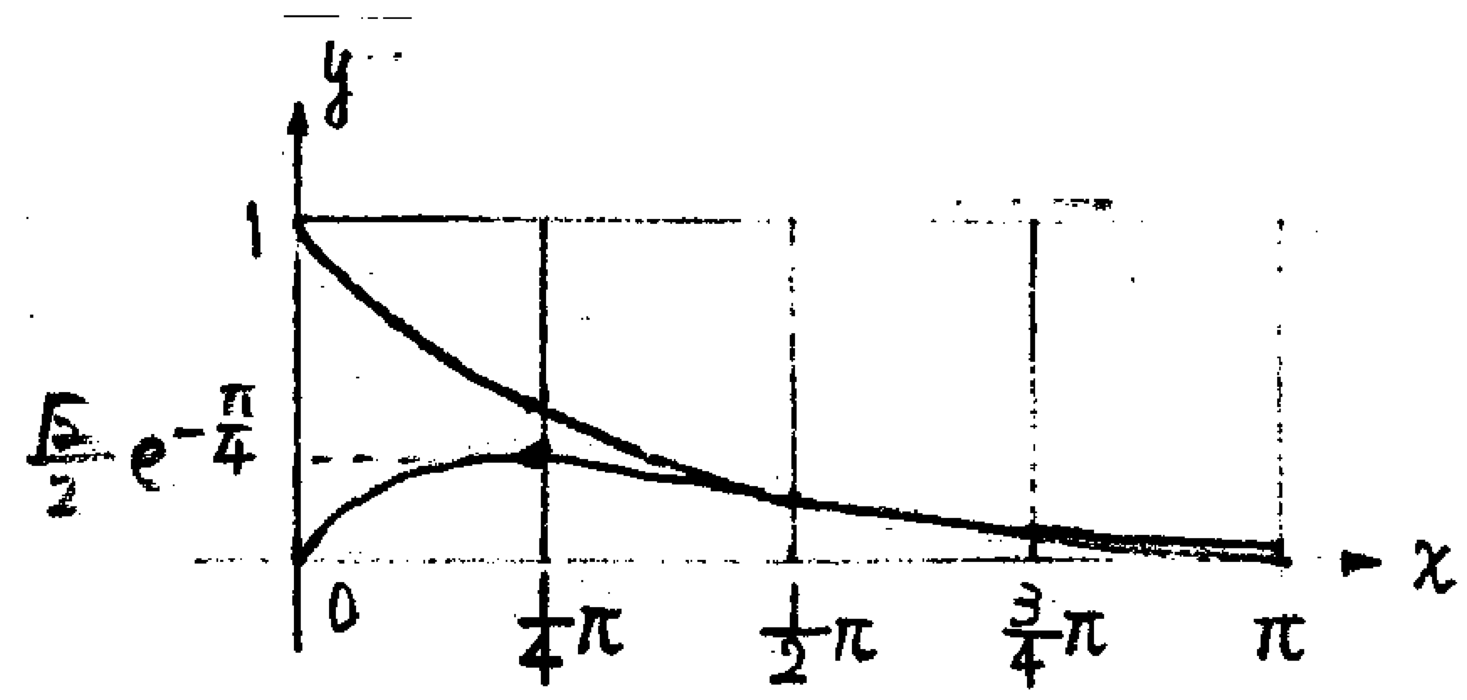
x	0	$\cdots$	$\frac{\pi}{4}$	$\cdots$	π
y'		$+$	0	$-$	
y	0	↗	極大	↘	0

$x = \frac{\pi}{4}$ で

$y = e^{-\frac{\pi}{4}}\sin\frac{\pi}{4}$

$= \boxed{\frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}}$

よってグラフは右図



(2) $I_n = \int_0^{n\pi} e^{-x}|\sin x|\,dx$

$= \sum_{k=0}^{n-1}\int_{k\pi}^{(k+1)\pi} e^{-x}|\sin x|\,dx$

$= \sum_{k=0}^{n-1}\int_{k\pi}^{(k+1)\pi} (-1)^k e^{-x}\sin x\,dx$

$= \sum_{k=0}^{n-1}(-1)^k\int_{k\pi}^{(k+1)\pi} e^{-x}\sin x\,dx$ ↲

ここで

$\int_{k\pi}^{(k+1)\pi} e^{-x}\sin x\,dx = I$ とおくと

$I = \int_{k\pi}^{(k+1)\pi}(-e^{-x})'\sin x\,dx$

$= \left[-e^{-x}\sin x\right]_{k\pi}^{(k+1)\pi} + \int_{k\pi}^{(k+1)\pi} e^{-x}\cos x\,dx$

$= \int_{k\pi}^{(k+1)\pi}(-e^{-x})'\cos x\,dx$

$= \left[-e^{-x}\cos x\right]_{k\pi}^{(k+1)\pi} - \int_{k\pi}^{(k+1)\pi} e^{-x}\sin x\,dx$

$= -e^{-(k+1)\pi}\cos(k+1)\pi + e^{-k\pi}\cos k\pi - I$

よって $\boxed{I = \frac{1}{2}\left(e^{-k\pi}\cos k\pi - e^{-(k+1)\pi}\cos(k+1)\pi\right)}$

No.

Date

ゆえに $I_n = \sum_{k=0}^{n-1} (-1)^k \times \frac{1}{2} \left(e^{-k\pi} \cos k\pi - e^{-(k+1)\pi} \cos (k+1)\pi \right)$

$$= \frac{1}{2} \left\{ (e^{-0} + e^{-\pi}) - (-e^{-\pi} - e^{-2\pi}) + (e^{-2\pi} + e^{-3\pi}) + (e^{-(n-1)\pi} + e^{-n\pi}) \right\}$$

$$= \frac{1}{2} \left\{ 1 + 2(e^{-\pi} + e^{-2\pi} + \cdots + e^{-(n-1)\pi}) + e^{-n\pi} \right\}$$

$$= \frac{1}{2} \left\{ 1 + 2 \times \frac{e^{-\pi}(1 - (e^{-\pi})^{n-1})}{1 - e^{-\pi}} + e^{-n\pi} \right\}$$

よって $\lim_{n \to \infty} I_n = \frac{1}{2} \left\{ 1 + \frac{2e^{-\pi}}{1 - e^{-\pi}} \right\}$ $\qquad \frac{1}{e^{n\pi}}$

$$= \frac{1}{2} \left\{ 1 + \frac{2}{e^{\pi} - 1} \right\}$$

$$= \frac{1}{2} \left\{ \frac{e^{\pi} - 1 + 2}{e^{\pi} - 1} \right\}$$

$$= \boxed{\frac{1}{2} \times \frac{e^{\pi} + 1}{e^{\pi} - 1}}$$