$f(x) = \int_x^{x+1} |\log t| \, dt$

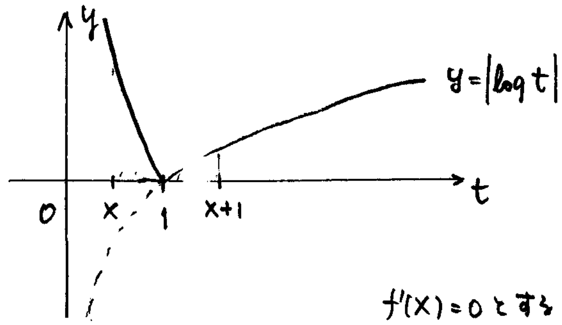
$G'(t) = g(t) = |\log t|$ とおくと

$f(x) = [G(t)]_x^{x+1}$

$f(x) = G(x+1) - G(x)$

よって $\boxed{f'(x)} = g(x+1) - g(x)$

$= \boxed{|\log(x+1)| - |\log x|}$



i) $0 < x < 1$ のとき

$f'(x) = \log(x+1) + \log x$

$= \log x(x+1)$

$f'(x) = 0$ とすると. $x(x+1) = 1$ より

$x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$\boxed{0 < x < 1 \text{ より} \quad x = \frac{-1+\sqrt{5}}{2}}$

ii) $x \geqq 1$ のとき

$f'(x) = \log(x+1) - \log x$

$= \log \frac{x+1}{x} > 0$

よって 増減表は.

x	0	…	$\frac{-1+\sqrt{5}}{2}$	…	1	…
$f'(x)$		$-$	0	$+$		$+$
$f(x)$		↘	極小	↗		↗

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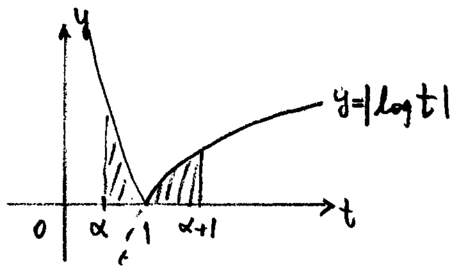
よって $f(x) = \int_x^{x+1} |\log t|\,dt$ は $x = \dfrac{-1+\sqrt{5}}{2}$ のとき
最小値をとる

$\alpha = \dfrac{-1+\sqrt{5}}{2}$ とおくと



$$f(\alpha) = \int_\alpha^{\alpha+1} |\log t|\,dt$$

$$= -\int_\alpha^1 \log t\,dt + \int_1^{\alpha+1} \log t\,dt$$

$$= -\int_\alpha^1 (t)' \log t\,dt + \int_1^{\alpha+1} (t)' \log t\,dt$$

$$= -\left[t\log t\right]_\alpha^1 + \int_\alpha^1 dt + \left[t\log t\right]_1^{\alpha+1} - \int_1^{\alpha+1} dt$$

$$= \alpha\log\alpha + 1 - \alpha + (\alpha+1)\log(\alpha+1) - (\alpha+1) + 1$$

$$= \alpha\log\alpha + (\alpha+1)\log(\alpha+1) + 1 - 2\alpha$$

$$= \frac{-1+\sqrt{5}}{2}\log\frac{-1+\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2}\log\frac{1+\sqrt{5}}{2} + 1 - (-1+\sqrt{5})$$

$$= \frac{1}{2}\log\frac{2}{-1+\sqrt{5}} + \frac{1}{2}\log\frac{1+\sqrt{5}}{2}$$
$$+ \frac{\sqrt{5}}{2}\log\frac{-1+\sqrt{5}}{2} + \frac{\sqrt{5}}{2}\log\frac{1+\sqrt{5}}{2} + 2 - \sqrt{5}$$

$$= \frac{1}{2}\log\frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}}{2}\log\frac{5-1}{4} + 2 - \sqrt{5}$$

$$= \frac{1}{2}\log\frac{5+1+2\sqrt{5}}{4} + 2 - \sqrt{5}$$

$$= \log\frac{\sqrt{6+2\sqrt{5}}}{2} + 2 - \sqrt{5}$$

$$= \boxed{\log\frac{\sqrt{5}+1}{2} + 2 - \sqrt{5}}$$

よって 最小値は $x = \dfrac{-1+\sqrt{5}}{2}$ のとき $\log\dfrac{\sqrt{5}+1}{2} + 2 - \sqrt{5}$
最大値なし

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$x > 1$ で

$$f(x) = \int_x^{x+1} \log t \, dt$$

$$= \int_x^{x+1} (t)' \log t \, dt$$

$$= [t \log t]_x^{x+1} - \int_x^{x+1} dt$$

$$= (x+1)\log(x+1) - x\log x - 1$$

$$= x \log \frac{x+1}{x} + \log(x+1) - 1$$

$$= x \log\left(1+\frac{1}{x}\right) + \log(x+1) - 1$$

$$= \underbrace{\log\left(1+\frac{1}{x}\right)^x}_{\to e} + \log(x+1) - 1 \longrightarrow \infty \quad (x \to +\infty)$$

よって $\boxed{\text{最大値なし}}$