

$$\begin{aligned}
 I &= \int_{-\pi}^{\pi} (x - a \sin x - b \cos x)^2 dx \\
 &= \int_{-\pi}^{\pi} (x^2 + a^2 \sin^2 x + b^2 \cos^2 x - 2ax \sin x + 2ab \sin x \cos x - 2b x \cos x) dx \\
 &= 2 \int_0^{\pi} (x^2 + a^2 \sin^2 x + b^2 \cos^2 x - 2ax \sin x) dx
 \end{aligned}$$

$$\begin{aligned}
 \therefore \int_0^{\pi} \sin^2 x dx &= \int_0^{\pi} \frac{1 - \cos 2x}{2} dx \\
 &= \left[\frac{x}{2} - \frac{\cos 2x}{4} \right]_0^{\pi} \\
 &= \frac{\pi}{2} - 1 + 1 = \boxed{\frac{\pi}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \int_0^{\pi} \cos^2 x dx &= \int_0^{\pi} \frac{1 + \cos 2x}{2} dx \\
 &= \left[\frac{x}{2} + \frac{\cos 2x}{4} \right]_0^{\pi} \\
 &= \boxed{\frac{\pi}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \int_0^{\pi} x \sin x dx &= \int_0^{\pi} x (-\cos x)' dx \\
 &= [-x \cos x]_0^{\pi} + \int_0^{\pi} \cos x dx \\
 &= -\pi \times (-1) + [\sin x]_0^{\pi} \\
 &= \boxed{\pi} \quad \text{だから}
 \end{aligned}$$

$$I = 2 \int_0^{\pi} x^2 dx + 2a^2 \times \frac{\pi}{2} + 2b^2 \times \frac{\pi}{2} - 4a \times \pi$$

$$= 2 \times \frac{\pi^3}{3} + a^2 \pi + b^2 \pi - 4a \pi$$

$$= \pi(a-2)^2 + b^2 \pi + \frac{2\pi^3}{3} - 4\pi$$

ゆえに I は $a=2, b=0$ のとき
最小値 $\frac{2\pi^3}{3} - 4\pi$ をとる