$$\int (x^2+a^2)^{-\frac{3}{2}}dx$$

$$= \int \frac{1}{\sqrt{(x^2+a^2)^3}}dx$$

・$a \neq 0$のとき

$x = a\tan\theta$ とおくと

$$dx = \frac{a}{\cos^2\theta}, \quad x^2+a^2 = a^2(\tan^2\theta+1)$$
$$= a^2 \times \frac{1}{\cos^2\theta}$$

$$よって(与式) = \int \frac{\frac{a}{\cos^2\theta}d\theta}{\sqrt{\left(\frac{a^2}{\cos^2\theta}\right)^3}}$$

$$= \int \frac{\frac{a}{\cos^2\theta}}{\pm\frac{a^3}{\cos^3\theta}}d\theta = \pm\int \frac{\cos\theta}{a^2}d\theta$$

$$= \pm\frac{1}{a^2}\sin\theta + C$$

ここで $\tan\theta = \frac{x}{a}$ より

$$1 + \frac{x^2}{a^2} = \frac{1}{\cos^2\theta}$$

$$\cos^2\theta = \frac{a^2}{a^2+x^2} \rightarrow \cos\theta = \frac{\pm a}{\sqrt{a^2+x^2}}$$

$$\sin\theta = \cos\theta\tan\theta$$
$$= \pm\frac{x}{\sqrt{a^2+x^2}}$$

$$ゆえに \boxed{(与式) = \frac{x}{a\sqrt{a^2+x^2}} + C}$$

$a = 0$のとき $\int x^{-3}dx = \int \frac{1}{x^3}dx$

$$= -\frac{1}{2x^2} + C$$