

$$\int \frac{dx}{(3e^x - 5e^{-x} + 2)}$$

$$= \int \frac{e^x}{3e^{2x} + 2e^x - 5} dx$$

$$= \int \frac{e^x}{(e^x - 1)(3e^x + 5)} dx$$

$$\therefore \text{let } t = e^x \text{ then}$$

$$dt = e^x dx$$

$$(5\text{th}) = \int \frac{dt}{(t-1)(3t+5)}$$

$$= \frac{1}{8} \int \left(\frac{1}{t-1} - \frac{3}{3t+5} \right) dt$$

$$= \frac{1}{8} \left(\log |t-1| - \log |3t+5| \right) + C$$

$$= \frac{1}{8} \log \left| \frac{t-1}{3t+5} \right| + C$$

$$= \boxed{\frac{1}{8} \log \left| \frac{e^x - 1}{3e^x + 5} \right| + C}$$

$$\frac{1}{(t-1)(3t+5)} = \frac{A}{t-1} + \frac{B}{3t+5} \quad \text{then}$$

$$= \frac{(3A+B)t + (5A-B)}{(t-1)(3t+5)}$$

$$3A+B=0$$

$$+ \frac{5A-B=1}{8A=1}$$

$$8A=1$$

$$A = \frac{1}{8}$$

$$B = -3A = -\frac{3}{8}$$