

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |2\cos^2 X + 3\sin X| dX.$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (2\cos^2 X + 3\sin X) dX \\ - \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} (2\cos^2 X + 3\sin X) dX.$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos 2X + 3\sin X) dX \\ - \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} (1 + \cos 2X + 3\sin X) dX$$

$$= \left[X + \frac{\sin 2X}{2} - 3\cos X \right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \\ - \left[X + \frac{\sin 2X}{2} - 3\cos X \right]_{-\frac{\pi}{2}}^{-\frac{\pi}{6}}$$

$$= \left(\frac{\pi}{2} + \frac{\pi}{6} - \frac{1}{2} \sin\left(-\frac{\pi}{3}\right) + 3\cos\left(-\frac{\pi}{6}\right) \right)$$

$$- \left(-\frac{\pi}{6} + \frac{1}{2} \sin\left(-\frac{\pi}{3}\right) - 3\cos\left(-\frac{\pi}{6}\right) + \frac{\pi}{2} \right)$$

$$= \frac{\pi}{2} + \frac{\pi}{6} - \frac{1}{2} \times \left(-\frac{\sqrt{3}}{2}\right) + 3 \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} - \frac{1}{2} \times \left(-\frac{\sqrt{3}}{2}\right) + 3 \times \frac{\sqrt{3}}{2} - \frac{\pi}{2}$$

$$= 2 \times \frac{\pi}{6} + 2 \times \frac{\sqrt{3}}{4} + 2 \times \frac{3\sqrt{3}}{2}$$

$$= \frac{\pi}{3} + \frac{\sqrt{3}}{2} + 3\sqrt{3}$$

$$= \frac{2\pi + 3\sqrt{3} + 18\sqrt{3}}{6}$$

$$= \boxed{\frac{2\pi + 21\sqrt{3}}{6}}$$

$$2\cos^2 X + 3\sin X$$

$$= 2(1 - \sin^2 X) + 3\sin X$$

$$= -(2\sin^2 X - 3\sin X - 2)$$

$$= -(\sin X - 2)(2\sin X + 1) \text{ (1)}$$

$$2\cos^2 X + 3\sin X = \begin{cases} -(2\cos^2 X + 3\sin X) & (-\frac{\pi}{2} \leq X \leq -\frac{\pi}{6}) \\ 2\cos^2 X + 3\sin X & (-\frac{\pi}{6} \leq X \leq \frac{\pi}{2}) \end{cases}$$

