(1) $\displaystyle\int_0^1 \frac{x}{\sqrt{4-3x^2}}\,dx$

$\displaystyle = \int_0^1 \frac{x}{\sqrt{3\left(\frac{4}{3}-x^2\right)}}\,dx$

ここで $x = \frac{2}{\sqrt{3}}\sin\theta$ とおくと

$dx = \frac{2}{\sqrt{3}}\cos\theta \cdot d\theta$

また $\frac{4}{3} - x^2 = \frac{4}{3} - \frac{4}{3}\sin^2\theta$

$= \frac{4}{3}\cos^2\theta$

x	$0 \cdots 1$
θ	$0 \cdots \frac{\pi}{3}$

ゆえに (与式) $\displaystyle = \int_0^{\frac{\pi}{3}} \frac{\frac{2}{\sqrt{3}}\sin\theta \times \frac{2}{\sqrt{3}}\cos\theta}{\sqrt{3\times\frac{4}{3}\cos^2\theta}}\,d\theta$ ($0<\theta<\frac{\pi}{3}$ より $\cos\theta>0$)

$\displaystyle = \int_0^{\frac{\pi}{3}} \frac{\frac{4}{3}\cos\theta\sin\theta}{2\cos\theta}\,d\theta$

$\displaystyle = \int_0^{\frac{\pi}{3}} \frac{2}{3}\sin\theta\, d\theta$

$= \frac{2}{3}\left[-\cos\theta\right]_0^{\frac{\pi}{3}}$

$= \frac{2}{3}\times\left(-\frac{1}{2}+1\right)$

$= \frac{2}{3}\times\frac{1}{2} = \boxed{\frac{1}{3}}$

別解 $t = \sqrt{4-3x^2}$ とおくと

$dt = \frac{-6x}{2\sqrt{4-3x^2}}\,dx$

$dt = \frac{-3x}{t}\,dx$

x	$0 \cdots 1$
t	$2 \cdots 1$

$x\,dx = -\frac{t}{3}\,dt$

ゆえに (与式) $\displaystyle = \int_2^1 \frac{1}{t}\times\left(-\frac{t}{3}\,dt\right)$

$\displaystyle = -\frac{1}{3}\int_2^1 dt$

$= -\frac{1}{3}\left[t\right]_2^1$

$= -\frac{1}{3}(1-2)$

$= \boxed{\frac{1}{3}}$