

$$\int \frac{(\log x)^\alpha}{x} dx$$

$$= \boxed{\frac{(\log x)^{\alpha+1}}{\alpha+1} + C} \quad (C \text{ は積分定数})$$

$$\int \frac{(\log x)^\alpha}{x} \log(\log x) dx$$

$$= \int \left( \frac{(\log x)^{\alpha+1}}{\alpha+1} \right)' \log(\log x) dx$$

$$= \frac{(\log x)^{\alpha+1}}{\alpha+1} \log(\log x) - \int \frac{(\log x)^{\alpha+1}}{\alpha+1} \times \frac{1}{\log x} \times \frac{1}{x} dx$$

$$= \frac{(\log x)^{\alpha+1}}{\alpha+1} \log(\log x) - \frac{1}{\alpha+1} \int \frac{(\log x)^\alpha}{x} dx$$

$$= \frac{(\log x)^{\alpha+1}}{\alpha+1} \log(\log x) - \frac{(\log x)^{\alpha+1}}{(\alpha+1)^2} + C$$

$$= \boxed{\frac{(\log x)^{\alpha+1}}{\alpha+1} \left\{ \log(\log x) - \frac{1}{\alpha+1} \right\} + C}$$